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Formulation and Analysis of Aircraft Steering Course

Audu Abdulkadir Iyyaka
Department of Computer Engineering, University of Maiduguri, Nigeria
Email: lm324fairchild@gmail.com

Abstract: *Although extensively studied in literature, due to its complexity navigational air current is still subject of intensive investigation. The key problem associated with the measurement of the characteristics of air current is that invariably both magnitude and direction may not be constant. These have led to problems when computing the typically complicated flight path which is commonly required in optimal time of flight analysis and calculation. This paper establishes which travel path between two points will provide the minimum transit time for a light aircraft which travels with a constant speed with respect to varying air current, made possible by the recent developments in the understanding of variational calculus.*

Keyword: *System dynamics; perturbation; constrained optimization; time of flight; variational calculus.*

1. INTRODUCTION

The recent developments in the understanding of flight formation of many birds have revealed that optimal spacing and group communication could most probably reduce the demand for flight power and energy expenditure, and improved orientation respectively [1]. With regards to aerodynamics, the flight of space vehicles is closely related to the phenomenon of bird flight.

While propulsion capability is a critical aspect of control systems which exhibit motion through fluid, the presence of friction related effects is a concern especially in aviation applications where the service integrity of aircrafts is a fundamental requirement to avoid longer travel time and to full safety standards.

The study and understanding of aerodynamics in terms of modeling aspect and computational efficiency of fluid-structure interaction (FSI) in several fields has been continuously growing in the last few years with the increasing need for the development of systems able to predict continuously the integrity of complex structures [2], [3], [4], [5], [6]. In order to be competitive with conventional non-conformal mesh evaluation techniques, accurate and stable coupling technique for fast-varying dynamics is being considered [2].

The flight of an aircraft is a time-evolving critical control process [7], [8], [9], [10], [11]. The structural stability of an aircraft, which is a dynamic system, in terms of its flight path, is highly dependent on the effect of small perturbation due to air current magnitude and direction [12].

A control system such as an aircraft is expected to be structurally stable such that it remains qualitatively the same in dynamics when subjected to small perturbations [12], [13], [14]. The characterization of perturbations has revealed that changes in the external parameters of an aircraft such as wind speed should not be significant enough to distort the structural stability of an aircraft in terms of its flight path, and that there should be no emergence of qualitatively new features such as eddy.

The pole-placement design technique of control system is widely regarded as capable of significantly improving the transient performance. Based on this technique, it is revealed that the transient response of an aircraft varies considerably with changes in airspeed and altitude. This means that the roots of the characteristic equation, and thus the location in the complex plane, of the poles and zeros of the aircraft transfer functions change widely. Thus, the root locus for each flight condition and the optimum gain for each loop vary [15].

For this reason, the ability to use Mach number as an independent variable of gains of all autopilots is extremely valuable during the flight regime as it potentially allows for the acquisition of stability derivatives with superior sensitivity and selectivity.

This work is motivated by the variational calculus formulation and analysis of aircraft steering course under the influence of air navigation current.

The organization of the paper is as follows. Section 2 details the aircraft dynamics. Constrained optimization is discussed in Section 3. Section 4 presents the derivation of time of flight along a steering course based on variational calculus approach. Discussion of the approach is made in section 5. Concluding statements is presented in Section 6.

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2. AIRCRAFT DYNAMIC

The flight of aircrafts through the atmosphere is often characterized by aerodynamic forces and moments [16]. This is depicted in Figure 1. $u(t)$ is input vector, $v(t)$ is vector which constitute perturbations, $x(t)$ is the output vector.

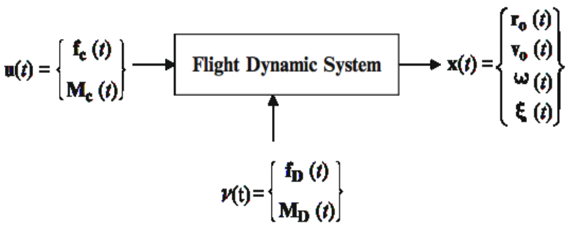

Figure 1 A typical flight dynamic system [16]

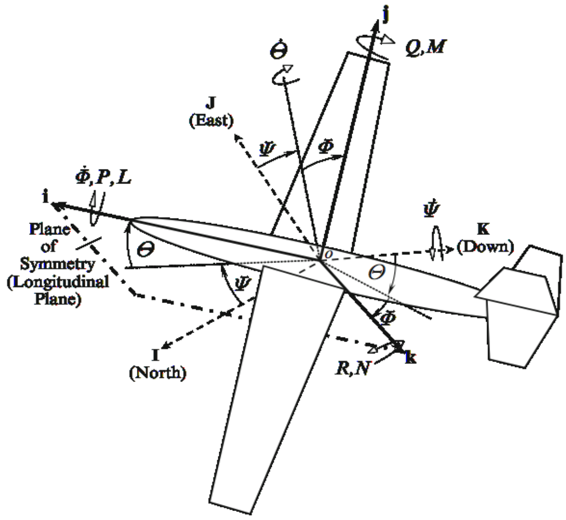

Figure 2 Plane of symmetry (the longitudinal plane) and the longitudinal motion, $\Phi = P = R = V = 0$ [16]

From Figure 2, the vector sum of aerodynamic force F_a and thrust T is expressed in (1)

$$F_a + T = Xi + Yj + Zk \quad (1)$$

The resolution of the force of gravity is expressed in (2) where m is the centre of mass of the aircraft and g is the acceleration due to gravity.

$$mg = mg(-\sin\Theta i + \sin\Phi\cos\Theta j + \cos\Phi\cos\Theta k) \quad (2)$$

The velocity of the center of mass, v(t), in (3) is taken to be the aircraft's linear velocity relative to the atmosphere.

$$\mathrm{v} = Ui + Vj + Wk \quad (3)$$

The total external force F on an aircraft is

$$F = m\frac{d\mathrm{v}}{dt} = m\dot{\mathrm{v}} + m(\omega \times \mathrm{v}) \quad (4)$$

$$X - mg\sin\Theta = m(\dot{U} + QW - RV) \quad (5)$$

$$Y + mg\sin\Phi\cos\Theta = m(\dot{V} + RU - PW) \quad (6)$$

$$Z + mg\cos\Phi\cos\Theta = m(\dot{W} + PV - QU) \quad (7)$$

3. CONSTAINED OPTIMIZATION

In countless scientific and control engineering applications, there often exit problems in which decision variables are restricted. The performance objective of a control system is usually expressed as an objective function. The hard objective function which must be satisfied before the soft objective function is satisfied is referred to as constrained optimization.

A nonlinear real-valued inequality function of the decision variable vector p in (8) can be used to describe constraints [17].

$$\phi(p) \leq \varepsilon \quad (8)$$

Where ε is a constant. In some control problems, the inequality may be required to be strict.

Constrained optimization often involves minimizing a scalar function $\phi_o(p)$ of some decision variable vector in a universal set U, subject to n conditions expressed as a function of p. A control problem is eventually formulated as

$$\min_{p \epsilon U} \phi_o(p) \quad (9)$$

Subject to

$$\phi_i \leq \varepsilon_i \quad \text{for } i = 1, 2, \ldots., n \quad (10)$$

In practice, all the constraints may not necessarily be satisfied. Based on the degree of violation such situations warrant that the constraints be relaxed and reformulated. Using variational calculus, it is possible to formulate the necessary optimization of a control problem. This approach is used in this work to derive the time of flight with respect to the steering course.

4. DERIVATION OF TIME OF FLIGHT

In calculus, the lengthds of an elemental arc described in a x-y plane can be expressed as

$$ds = \left(1 + y'(x)\right)^{\frac{1}{2}} \quad (11)$$

For an arc spanning between two points where, $x \,\epsilon\, [0, x_1]$, the total arc length will be as expressed in (12).

$$s = s(x) = \int_o^x \left(1 + y'(x)\right)^{\frac{1}{2}} dx \quad \text{for } x \,\epsilon\, [0, x_1] \quad (12)$$

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Through pure kinematics, the time required to travel the arc length joining two points A and B is given as

$$T = T(y) = \int_0^{AB} \frac{ds}{v(x)} \qquad (13)$$

where $v(x) = v$, is the speed of travel. Substituting for ds from (11) in (13), we have

$$T = T(y) = \int_0^{x_1} \frac{\left(1+y^{'}(x)\right)^{\frac{1}{2}}}{v(x)} dx \qquad (14)$$

Consider a light plane aircraft travelling at a constant speed v from point A to B in space as depicted in Figure 1. Also, it is assumed that the air current r is directed in the y axis directionwith the consideration that $r = r(x)$, continuous on $[0,\ x_1]$. From Figure 3, we can write

$$y = x \tan\sigma \qquad (15)$$

$$\frac{dy}{dx} = y^{'}(x) = \tan\sigma \qquad (16)$$

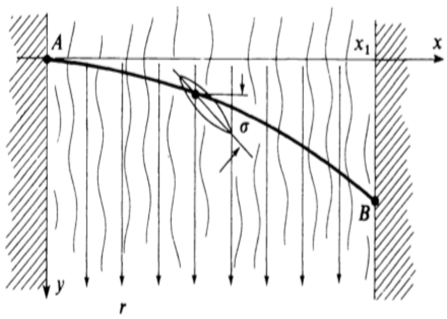

Figure 3 Path description of light aircraft [18]

Therefore, (14) becomes

$$T = T(y) = \int_0^{x_1} \frac{\left(1+(\tan\sigma)^2\right)^{\frac{1}{2}}}{v(x)} dx \qquad (17)$$

However, trigonometric identity admits that

$$(\sin\sigma)^2 + (\cos\sigma)^2 = 1 \qquad (18)$$

From (18), it can be written that

$$(\tan\sigma)^2 + 1 = (\sec\sigma)^2 \qquad (19)$$

Simplifying (7) further we get

$$T = T(y) = \int_0^{x_1} \frac{\sec\sigma}{v} dx \qquad (20)$$

Equation (20) provides the expression for the computation of time of travel from A to B.

The component $v_x(x)$ and $v_y(x)$ of v in the x-axis and y-axis directions are given in (11) and (12) respectively

$$v_x = v\cos\sigma \qquad (21)$$

$$v_y = r(x) + v\sin\sigma \qquad (22)$$

The distance$x$ and$y$ travelled in a second in the x-axis and y-axis direction respectively are

$$x = v\cos\sigma \qquad (23)$$

$$y = r(x) + v\sqrt{1-(\cos\sigma)^2} \qquad (24)$$

$$y(x) = r(x) + v\cos\sigma\sqrt{\frac{1}{(\cos\sigma)^2} - 1} \qquad (25)$$

$$y(x) = \left[\frac{r(x)}{v\cos\sigma} + \sqrt{(\sec\sigma)^2 - 1}\right] v\cos\sigma \qquad (26)$$

$$y(x) = \left[\frac{r(x)}{v}\sec\sigma + \sqrt{(\sec\sigma)^2 - 1}\right] v\cos\sigma \qquad (27)$$

$$y(x) = \left[\frac{r(x)}{v}\sec\sigma + \sqrt{(\sec\sigma)^2 - 1}\right] x \qquad (28)$$

The x rate of change of y, that is, $y^{'}(x)$ can be written as

$$y^{'}(x) = \left[\frac{r(x)}{v}\sec\sigma + \sqrt{(\sec\sigma)^2 - 1}\right] + M \qquad (29)$$

$$M = x\left(\frac{\sec\sigma}{v}\right) r^{'}(x) = r^{'}(x) \qquad (30)$$

It is strictly required to have$0 \le r(x) < 1$,in order that the crossing be possible. Therefore, $r^{'} \ll 0$. Hence (29) simplifies to

$$y^{'}(x) = \left[\frac{r(x)}{v}\sec\sigma + \sqrt{(\sec\sigma)^2 - 1}\right] \qquad (31)$$

From (21), it is possible to write

$$\frac{v}{v_x} = \frac{1}{\cos\sigma} = \frac{1}{\sqrt{1-(\sin\sigma)^2}} \qquad (32)$$

$$\sec\sigma = \frac{1}{\sqrt{1-(\sin\sigma)^2}} \qquad (33)$$

$$\sec\sigma = \frac{1}{\sqrt{1-(y^{'}-r)^2}} \qquad (34)$$

$$\sec\sigma = \frac{1}{\sqrt{1-(y^{'})^2 + 2ry^{'} - r^2}} \qquad (35)$$

$$\sec\sigma = \frac{1}{\left[\sqrt{1-r^2}\right]\left[\sqrt{1-\frac{(y^{'})^2}{1-r^2}+\frac{2ry^{'}}{1-r^2}}\right]} \qquad (36)$$

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$$\frac{1}{1-r^2} = -\beta^2 \quad (37)$$

$$A = \beta\sqrt{-1}B \quad (38)$$

Where

$$A = \sec\sigma \quad (39)$$

$$B = \left(\frac{1}{\left[\sqrt{1+\beta^2(y^{'})^2}\right]\left[\sqrt{1-\frac{2r\beta^2 y^{'}}{1+\beta^2(y^{'})^2}}\right]}\right) \quad (40)$$

$$A = \beta\sqrt{-1}\left([1+\beta^2(y^{'})^2]^{-\frac{1}{2}}\right)\left(\left(1 - \frac{2r\beta^2 y^{'}}{1+\beta^2(y^{'})^2}\right)^{-\frac{1}{2}}\right) \quad (41)$$

$$-\frac{1}{2} = \frac{1}{2}(-1+0) = (-1+0)^{\frac{1}{2}} = \sqrt{-1} \quad (42)$$

If (42) is true, then (41) becomes

$$A = \beta\sqrt{1+\beta^2(y^{'})^2}\left(1 - \frac{2r\beta^2 y^{'}}{1+\beta^2(y^{'})^2}\right) \quad (43)$$

$$A = \beta\sqrt{1+\beta^2(y^{'})^2}\left(1 - \frac{r\beta^2 y^{'}}{\frac{1}{2}(1+\beta^2(y^{'})^2)}\right) \quad (44)$$

$$A = \beta\sqrt{1+\beta^2(y^{'})^2}\left(1 - \frac{r\beta^2 y^{'}}{\beta\sqrt{1+\beta^2(y^{'})^2}}\right) \quad (45)$$

$$A = \beta\sqrt{1+\beta^2(y^{'})^2} - r\beta^2 y^{'} \quad (46)$$

Therefore, (20) becomes

$$T = \int_0^{x_1}\frac{1}{v}\left[\beta\sqrt{1+\beta^2(y^{'})^2} - r\beta^2 y^{'}\right]dx \quad (47)$$

The Gâteaux variation of Tat y in the direction v is defined as

$$\delta T(y;v) \stackrel{\text{def}}{=} \lim_{\varepsilon\to 0}\frac{T(y+\varepsilon v)-T(y)}{\varepsilon} \quad (48)$$

The existence of the limit presupposes [18] that

$$T(y) \text{ is defined;} \quad (50)$$

$$\delta T(y;v) = \frac{\partial}{\partial x}T(y+\varepsilon v)\Big|_{\varepsilon=0} \quad (51)$$

$$\frac{\partial}{\partial x}T(y+\varepsilon v) = \int_0^{x_1}\frac{\partial}{\partial \varepsilon}\left(\frac{1}{v}\left[\beta\sqrt{1+\beta^2((y+\varepsilon v)^{'})^2} - r\beta^2(y+\varepsilon v)^{'}\right]\right)dx \quad (52)$$

$$\frac{\partial}{\partial x}T(y+\varepsilon v) = \int_0^{x_1}\left[\frac{\beta}{v}\frac{\beta^2(y+\varepsilon v)^{'}(x)v^{'}(x)}{\sqrt{1+\beta^2((y+\varepsilon v)^2)^1}} - \frac{r\beta^2 v^{'}}{v}\right]dx \quad (53)$$

Equation (53) now evaluated at $\varepsilon = 0$. This yield

$$\delta T(y;v) = \int_0^{x_1}\left[\frac{\beta}{v}\frac{\beta^2(y)^{'}(x)v^{'}(x)}{\sqrt{1+\beta^2((y)^2)^1}} - \frac{r\beta^2 v^{'}}{v}\right]dx \quad (54)$$

5. DISCUSSION

The analysis of time of travel of aircrafts through varying air current is a sphere of influence of flight control engineering. Having been the focal point of this work, variational calculus technique is firstly used to seek path description and distance of travel of the aircraft. This approach provides opportunity to proceed to determine an optimal time of flight expression (54) of the aircraft based on Gâteaux variation. Equation (47) is the objective function of the aircraft control system.

6. CONCLUSION

This study demonstrates the suitability of variational calculus to accurately and objectively optimize the steering course of an aircraft which is considered as a control system. An aircraft flight is previewed as a process which is significantly affected by the FSI phenomenon. The analysis enables both the objective function related to the aircraft steering course to be determined and consequently used to derive an expression for the optimal time of flight expression of the aircraft. Also, the analysis presented has possible applications in aiding autopilot development and in the realistic modeling of aircraft dynamic studies and design verification testing.

Finally, the method presented can be used to characterize flight path geometry which provides capabilities for computation of the extent of aircraft motion and the influence of air on the aircraft dynamic

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Authors Biography

Audu Abdulkadir Iyyaka, is a Lecturer in University of Maiduguri, Nigeria. He completed his B.Eng./ M.Eng. in Electrical and Electronic Engineering and Electronic Engineering respectively at Abubakar Tafawa Balewa University Nigeria. His research interests are artificial intelligence, embedded systems, electronic devices, communication systems, control systems, and signal and image processing.

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