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Dynamic Optimization of Time of Flight of Rocket Missile System

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Abstract: *The nonalignment of a rocket with the velocity vector can lead to an increase in the time of flight or unsuccessful flight. This theoretical study employs Lagrange method, which is a dual method of constrained optimization to derive an expression for the optimal time of flight of rocket-missile system (RMS).*

Keyword: *Damped motion; system dynamics; constrained optimization; time of flight; variational calculus.*

1.INTRODUCTION

Advancements in the design and fabrication of mechanical systems, understanding of fuel chemistry, and computer technology have led to several sophisticated developments currently associated with RMS. Each development is associated with a different degree of complexity and associated computational and temporal cost.

Interestingly, dynamic targets have become an important aspect of modern warfare. Such targets are characterized with high mobility and extensive depth among other things. For this reason, there is a need for an extensive improvement in missile technology. In the view of [1], such an improvement is expected to feature, considerable increase in missile range, accuracy, and highly destructive capability.

Second World War and in some cases even to the First World War was the starting era of the concept of precision guided missiles. This idea which is now being explored further is another factor that has both gained media attention, in view of its surgical precision and played significant role in recent conflicts [2].

From technical point of view, changes in missile velocity, direction, and centre of mass coordinates are often consequences of deliberately created aerodynamic forces and moments as a result of local structural deformation of the missile [1]. This concept is shown in Figure 1. The control of trajectory is achieved by changing the missile's aerodynamic characteristics and ballistic trajectory. Trajectory control can improve the missile's target accuracy and range capability.

In this paper, missile time of flight optimization is demonstrated with particular emphasis on rocket thrust. It provides another dimension through which the efficient improvement of missile performance in the presence of thrust uncertainty can be studied. RMS is a critical control system [3], [4], [5], [6]. During flight, a RMS is subjected to random inputs which do not allow the output to be strictly within prescribed bounds. In flight control systems, performance objectives are described by a set of objective functions.

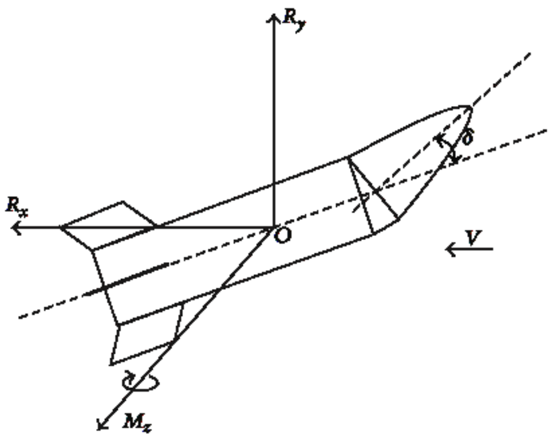

Figure 1 Diagram for simplifying aerodynamic forces. [2]

The design problem formulated as a set of inequalities includes the output performance criteria in the time domain and the robust performance criterion in the frequency domain.

The key problem associated with aerodynamic load is that the rocket is required to be continuously aligned with the velocity vector [7]. For a rocket filled with liquid propellants, any significant transverse acceleration due to lift (or sideforce) can prove to be destructive. However, ideas that have been investigated indicate that rocket axisymmetry in shape and mass distribution; make possible an equilibrium flight condition without a net lift and sideforce.

In order to correct attitude deviations from equilib-

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rium, most rockets are also equipped with an attitude control system for making the vehicle follow the desired flight path, such that angle-of-attack and sideslip angle are always zero. Such a control system applies its control forces and moments through aerodynamic control surfaces and thrust deflection [7].

Rocket maneuvers are performed by generating a pure torque (without a net lateral force), which causes the large rocket thrust to act in a new direction. Smaller rockets – such as air-to-air missiles – have movable fins mounted fore and aft of the center of mass in a way similar to aircraft lifting surfaces. However, the fins are much smaller than the wings and tails of an aircraft, because they are intended.

This paper is organized as follows. Section 2 briefly presents the concept of thrust in rockets. In Section 3, thrust vectoring is explained. Section 4 presents rocket damped-motion. Brief discussion is made is in Section 5. Section 6 presents final remarks.

2. THRUST AND ITS EFFECTIVENESS

High thrust is a critical requirement in the design of rocket system. It affects both the propulsion system design and the trajectory performance. It is observed that a compromise must be sought, as greater thrust levels reduce the gravitational losses but increase structural mass [8]. Despite the low regression rate of the fuel, motor designs of larger diameters that could produce high thrust suitable for space launch vehicles. [9].

With a highly accurate dimensionality of the nozzle, a rocket missile system will move at a high speed as a result of high thrust developed by the exhaust gases leaving the nozzle section [10]. It is stated in [11] that in pulse detonation engine (PDE), thrust is achieved through intermittent detonation.

3. OPTIMALITY OF MISSILE PATH

Path optimization can be adopted to account for the unpredictable challenges during missile motion. In one of the commonly encountered situations, an external sensor to the missile is used to acquire critical information about a target [1], [2]. This information is then used to guide the missile during its course towards the target. This approach finds significant application long range missiles.

Another strong aspect of optimization during mid-course is identified, effectively taken advantage of, and broadly employed in air-to-air missiles [2]. In this area of application, the lower drag at higher altitude and its ability to greatly improve path travel is broadly assessed. With mid-course optimization, minimal time of flight, maximal range, maximal terminal velocity etc can be significantly improved upon.

Special emphasis is paid to the description and application of angle of impact, no angle of attack at impact adjustments in air-to-surface missiles. This ensures that the response and performance of a penetrating warhead, particularly in the last lap of missile travel, is representative of the characteristics of a given target profile through terminal trajectory optimization [2].

Randomized route assessment and planning can be a major focus in which the range of applicability of trajectory optimization can be improved. The uncertainty in the different missile path and their effect on the missile is also quantified to avoid obstacles, maximize survivability, maximize the probability of target acquisition etc [2]. As a result, a path dependent adjustment factors are developed which can be used to achieve an alternative appropriate path to approach the target [2]. This planning is normally done prior to launch, consequently, through progressively more complicated parametric analyses re-planning can be done in flight by the missile. The potential bias introduced through computational power and optimization algorithms have now made it possible to use real time optimization in flight.

4. THRUST VECTORING

The flight of aircrafts through the atmosphere is often characterized by aerodynamic forces and moments [12].

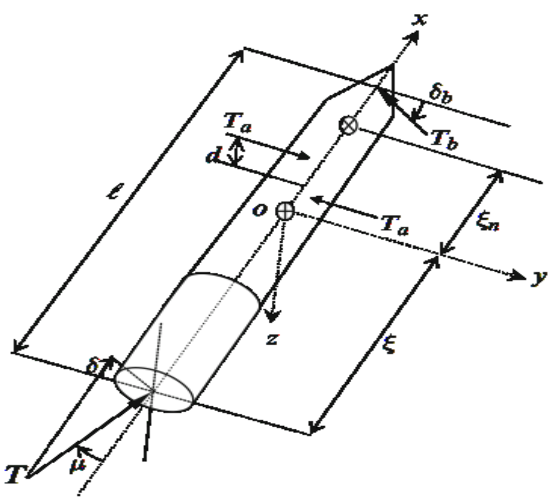

Figure 2 Thrust-vectoring geometry for a rocket [7]

Control torque for attitude can be generated by thrust vectoring [7]. It involves equilibrium realization through the use of transverse forces. Conceptually, at least two equal and opposite control forces is necessary to form a pure couple. Vectored thrust could be opposed by control force generated by aerodynamic fin. Such an arrangement is commonly found in short-range missiles. For launch vehicle, thrust-vectoring can be realized in several ways such as gimbaling of a single main engine balanced by reaction jets or vernier rockets. This is depicted in Figure 2.

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5. DAMPED ROCKET MOTION

In this analysis, a rocket of mass m is considered to be accelerated from rest at the earth's surface at an angle β to the horizontal, to a height H in time T_H, by the thrust ma of its engine. Where, a, is the rocket acceleration. Assuming the fuel consumption, F, of the rocket is modeled as in (1), which is the objective function.

$$F(a) = \int_0^{T_H} a^2(t)dt \tag{1}$$

Also, supposing h is so small that both m and g, the acceleration due to gravity, remain constant during flight, then we wish to control the thrust to minimize the fuel consumption stated in (1).

When the time of flight is constant, then the effective acceleration of the rocket in the vertical direction can be written by using Newton's second law of rectilinear motion as

$$\frac{d^2s}{dt^2} = a\sin\beta - g \tag{2}$$

When the effect of atmospheric drag γ, is taken into consideration, (2) becomes

$$\frac{d^2s}{dt^2} = a\sin\beta - g - \gamma\frac{ds}{dt}, \quad \text{for } \gamma \geq 0 \tag{3}$$

Equation (3) can be manipulated as follows

$$\frac{d}{dt}\left(\frac{ds}{dt}\right) = a\sin\beta - g - \gamma\frac{d}{dt}(s) \tag{4}$$

$$\frac{d}{dt}\left[\left(\frac{ds}{dt}\right) + \gamma s\right] = a\sin\beta - g \tag{5}$$

The multiplication of (5) by $e^{-\gamma t}$ yields

$$\frac{d}{dt}\left[\left(\frac{ds}{dt}\right)e^{-\gamma t} - \gamma s e^{-\gamma t}\right] = (a\sin\beta - g)e^{-\gamma t} \tag{6}$$

$$\frac{d}{dt}\left[\frac{d}{dt}(se^{-\gamma t})\right] = (a\sin\beta - g)e^{-\gamma t} \tag{7}$$

Therefore,

$$\frac{d^2s}{dt^2} = (a\sin\beta - g)e^{-\gamma t} \tag{8}$$

Simplifying (3) further using (8), we have

$$\gamma\frac{ds}{dt} = (a\sin\beta - g) - (a\sin\beta - g)e^{-\gamma t} \tag{9}$$

Considering the time of flight $T_H - t$, (9) is written as

$$\gamma s = \int_0^{T_H}\left(1 - e^{-\gamma(T_H - t)}\right)(a\sin\beta - g)dt \tag{10}$$

The constrained function J is defined as

$$J = 2\gamma s = \int_0^{T_H} 2\left(1 - e^{-\gamma(T_H - t)}\right)(a\sin\beta - g)dt \tag{11}$$

Now, the minimization of (1) will be embarked upon subject to (11). This will be done by seeking a closed form solution of the Lagrange duality which essentially turns a constrained problem into an almost unconstrained one. On the basis of duality theory, the definition of Lagrange function is

$$L(a,\lambda) = F(a) + \sum_{j=1}^{l}\lambda_j g_j(a) \tag{12}$$

$$\lambda_j \leq 0$$

Equation (12) is true if there exits $\lambda = [\lambda_1\,\lambda_2\lambda_3\,...\,\lambda_j]$. F(a) is the objective function and g_j are the constrained functions. In this problem, there is only one objective function, that is, J.

Therefore, the Lagrange function $L(a,\lambda)$ of this problem is

$$L(a,\lambda) = a^2 - 2\lambda\left[\left(1 - e^{-\gamma(T_H - t)}\right)(a\sin\beta - g)\right] \tag{13}$$

$$\frac{\partial L(a,\lambda)}{\partial a} = 2a(t) - 2\lambda\left(1 - e^{-\gamma(T_H - t)}\right)\sin\beta = 0 \tag{14}$$

$$a_o(t) = \lambda_o\left(1 - e^{-\gamma(T_H - t)}\right)\sin\beta \tag{15}$$

$$N = \int_0^{T_H} 2\left(1 - e^{-\gamma(T_H - t)}\right)(a_o(t)\sin\beta - g)dt \tag{16}$$

The integration of (16) is

$$J = 2(\lambda_o(\sin\beta)^2 - g)t + \frac{2\lambda_o(\sin\beta)^2}{\gamma}e^{-\gamma(T_H - t)} - \frac{\lambda_o}{2\gamma}e^{-2\gamma(T_H - t)} - \frac{g}{\gamma}e^{-\gamma(T_H - t)} \tag{17}$$

$$J = 2(\lambda_o(\sin\beta)^2 - g)T_H - \frac{2\lambda_o(\sin\beta)^2}{\gamma}e^{-\gamma T_H} + \frac{\lambda_o}{2\gamma}e^{-2\gamma T_H} + \frac{g}{\gamma}e^{-\gamma T_H} = 2\gamma S \tag{18}$$

$$\lambda_o = \frac{2\gamma S - \frac{g}{\gamma}e^{-\gamma T_H} + 2gT_H}{2T_H(\sin\beta)^2 - \frac{2(\sin\beta)^2}{\gamma}e^{-\gamma T_H} + \frac{1}{2\gamma}e^{-2\gamma T_H}} \tag{19}$$

From (1),

$$F(a_o) = \lambda_o^2(\sin\beta)^2\int_0^{T_H}\left(1 - e^{-\gamma(T_H - t)}\right)^2 dt$$

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$$F(a_o) = \lambda_o^2 (\sin\beta)^2 \left[t + \frac{2}{\gamma} e^{-\gamma(T_H - t)} - \frac{1}{2\gamma} e^{-2\gamma(T_H - t)} \right]_0^{T_H} \quad (20)$$

$$F(a_o) = \lambda_o^2 (\sin\beta)^2 \left(T_H - \frac{2}{\gamma} e^{-\gamma T_H} + \frac{1}{2\gamma} e^{-2\gamma T_H} \right) \quad (21)$$

Differentiating (21) with respect to T_H, we have

$$F(a_o) = (\sin\beta)^2 \left(\frac{2\gamma S - \frac{g}{\gamma} e^{-\gamma T_H} + 2gT_H}{2T_H (\sin\beta)^2 - \frac{2(\sin\beta)^2}{\gamma} e^{-\gamma T_H} + \frac{1}{2\gamma} e^{-2\gamma T_H}} \right)^2 B + CD \quad (22)$$

$$B = 1 + 2e^{-\gamma T_H} - e^{-2\gamma T_H} \quad (23)$$

$$C = T_H - \frac{2}{\gamma} e^{-\gamma T_H} + \frac{1}{2\gamma} e^{-2\gamma T_H} \quad (24)$$

$$D = 2 \left(\frac{2\gamma S - \frac{g}{\gamma} e^{-\gamma T_H} + 2gT_H}{2T_H (\sin\beta)^2 - \frac{2(\sin\beta)^2}{\gamma} e^{-\gamma T_H} + \frac{1}{2\gamma} e^{-2\gamma T_H}} \right) E \quad (25)$$

$$E = \frac{(FG - HI)}{\left(2T_H (\sin\beta)^2 - \frac{2(\sin\beta)^2}{\gamma} e^{-\gamma T_H} + \frac{1}{2\gamma} e^{-2\gamma T_H} \right)^2} \quad (26)$$

$$F = 2T_H (\sin\beta)^2 - \frac{2(\sin\beta)^2}{\gamma} e^{-\gamma T_H} + \frac{1}{2\gamma} e^{-2\gamma T_H} \quad (27)$$

$$G = g e^{-\gamma T_H} + 2g \quad (28)$$

$$H = 2\gamma S - \frac{g}{\gamma} e^{-\gamma T_H} + 2gT_H \quad (29)$$

$$I = 2(\sin\beta)^2 + 2(\sin\beta)^2 e^{-\gamma T_H} - e^{-2\gamma T_H} \quad (30)$$

The minimization of (22) will yield the optimal time of flight expression, T_o.

6. DISCUSSION

Lagrange method [13], which is a dual method of constrained optimization widely discussed in variational calculus [14], is used to derive an expression for the optimal time of flight of RMS. The effect of rocket thrust and any further implications of using constrained variables are demonstrated through the formulation of constrained function assessment.

In this optimization analysis, a robust control which will allow for the maintenance of design specification, such as missile time of flight in the presence of rocket thrust uncertainty is closely quantified. The constrained function in this analysis is defined in (12).

Through progressively more complicated parametric analyses, it is observed that a Lagragian function is a concave function in Lamda which does not require any assumption about the convexity of the objective function or the constraint function. The optimal time of flight is dependent on g, S, γ and β.

7. CONCLUSION

The main conclusion of this work is that through variational calculus, it is easier to define the constrained function required in the derivation of optimal time of flight of RMS. The Lagragian method proves to be easier and translate the problem to an unconstrained optimization type. The significant of rocket thrust as a major uncertainty is considered. The analysis of the obtained optimal time of flight would be carried out in future work.

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